VIBEKA NAND JHA

ASSOCIATE PROFESSOR

DEPARTMENT OF PHYSICS

SNSRKS COLLEGE SAHARSA

PHYSICS (H)-PAPER I

1

Elasticity

Elasticity is that property of matter by Virtue of which a body regains its original shape or size on the removal of the deforming forces.

Types of Elasticity:- We have three types of Elasticity:-

(i) Young's Modulus (Y).
(ii) Bulk Modulus (K)
(iii) Modulus of rigidity (η).

Poisson's Ratio (σ) :-

Within elastic limit, the lateral strain is proportional to the longitudinal strain ie, the ratio of lateral strain and longitudinal strain is a constant for the material of a body. This constant is known as Poisson's ratio (σ). If β and α are the lateral and longitudinal strain respectively, then

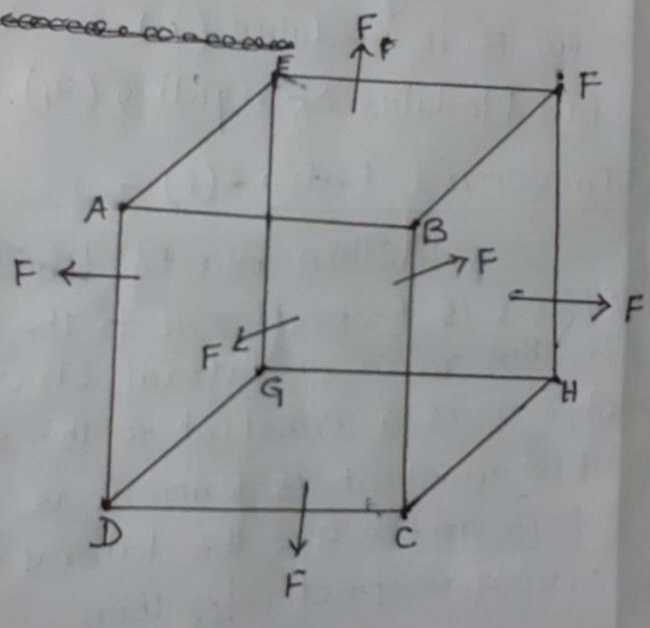
~~Poisson's~~

Poisson's Ratio (σ) = β/α

Relations between the Elastic Constants:

The elastic constants are dependent to each other, since any change in the size and shape of a body may be obtained by first changing the size of the body only and then by changing the shape only. Thus, the expressions can be derived, showing the inter-relations between them.

(1) Relations between Y, K and σ.



Let ABCDEFGH represent a cube of unit side. Let us consider a force F, which acts normally and uniformly on each of its six faces in the outward direction.

If α is the increase per unit length per unit tension along the direction of the force, then the elogation produced in each of the edges, namely AB, BF and BC, will be $F\alpha$.

If β is the contraction produced per unit length per unit tension perpendicular to the edges, then contraction produced ~~[illegible]~~ perpendicular to each of the edge, namely AB, BF and BC will be ~~[illegible]~~ $F\beta$.

Thus, the ~~[illegible]~~ sides of the cube become

$$AB = 1 + F\alpha - F\beta - F\beta$$
$$= 1 + F(\alpha - 2\beta)$$
$$BF = 1 + F\alpha - F\beta - F\beta.$$
$$= 1 + F(\alpha - 2\beta)$$
$$BC = 1 + F\alpha - F\beta - F\beta$$
$$= 1 + F(\alpha - 2\beta)$$

Hence, the final volume of the cube is

$$AB \times BF \times BC = [1 + F(\alpha - 2\beta)] \times [1 + F(\alpha - 2\beta)] \times [1 + F(\alpha - 2\beta)]$$
$$= [1 + F(\alpha - 2\beta)]^3$$
$$= [1 + 3F(\alpha - 2\beta)]$$ [According to ~~[illegible]~~ Binomial expansion]

Neglecting α and β since they are very small quantities.

$\therefore$ Change in Vol. = final Vol − initial Vol.

$$= 1 + 3F(\alpha - 2\beta) - 1$$
$$= 3F(\alpha - 2\beta)$$

Volume strain $= \dfrac{3F(\alpha - 2\beta)}{1} = 3F(\alpha - 2\beta)$

4.

Bulk Modulus (K) $= \dfrac{\text{Normal Stress}}{\text{Volume Strain}}$

$$= \frac{F}{3F(\alpha-2\beta)}$$

$$= \frac{1}{3(\alpha-2\beta)}$$

or, $K = \dfrac{1/\alpha}{3(1-2\beta/\alpha)}$

But α is the increase per unit length per unit tension hence stress = 1 as area of any edge of the cube is unity.

$\therefore\ Y = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{1}{\alpha}$ and $\sigma = \beta/\alpha$

Therefore, $\boxed{K = \dfrac{Y}{3(1-2\sigma)}}$ —— [A]