Differentiation of Vectors

THEOREM: The necessary and sufficient condition for the vector function $\vec{a}(t)$ to have constant magnitude is

$\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$ i.e. $\vec{a} \perp d\vec{a}$

or if $|\vec{a}(t)|$ be constant, prove that $\vec{a}$ and $\frac{d\vec{a}}{dt}$ are at right angle.

Proof: The condition is necessary:- we have to show that if the vector function $\vec{a}(t)$ is of constant magnitude then $\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$

Let $|\vec{a}(t)| = a = $ Constant

Then $\vec{a} \cdot \vec{a} = a^2 = $ Constant

$\therefore \frac{d}{dt}[\vec{a} \cdot \vec{a}] = \vec{a} \cdot \frac{d\vec{a}}{dt} + \frac{d\vec{a}}{dt} \cdot \vec{a}$

i.e. $\frac{d}{dt}(a^2) = 2\vec{a} \cdot \frac{d\vec{a}}{dt}$

i.e. [illegible] $\frac{d}{dt}$

i.e. [illegible] $\frac{d}{dt}$

Hence the condition is necessary. The condition is sufficient:-

We have to show that if $\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$ then the vector function $\vec{a}(t)$ is of constant magnitude.

$\because \vec{a} \cdot \frac{d\vec{a}}{dt} = 0 \Rightarrow \vec{a} \cdot \frac{d\vec{a}}{dt} + \frac{d\vec{a}}{dt} \cdot \vec{a} = 0$

i.e. $\frac{d}{dt}(\vec{a} \cdot \vec{a}) = 0$

i.e. $d(\vec{a} \cdot \vec{a}) = 0$

Integrating we have

$\vec{a} \cdot \vec{a} = $ Constant $\Rightarrow a^2 = $ Constant

$\Rightarrow a = $ Constant

or $|\vec{a}| = $ Constant

Hence the constant condition is sufficient.

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Th-3. Find the condition that the line $x\cos\alpha + y\sin\alpha = p$ may be normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Soln :- If the straight line $x\cos\alpha + y\sin\alpha = p$ be a normal to the given ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at φ, it must be identical with

$a x\sec\varphi - b y\,\mathrm{cosec}\,\varphi = a^2-b^2$

The condition for this is

$$\frac{\cos\alpha}{a\sec\varphi} = \frac{\sin\alpha}{-b\,\mathrm{cosec}\,\varphi} = \frac{p}{a^2-b^2}$$

Taking 1st and last relation, we get

$$ap\sec\varphi = (a^2-b^2)\cos\alpha$$

$$\frac{ap}{\cos\varphi} = (a^2-b^2)\cos\alpha$$

$$\therefore \cos\varphi = \frac{ap}{(a^2-b^2)\cos\alpha}$$

$$\therefore \cos^2\varphi = \frac{a^2p^2}{(a^2-b^2)^2\cos^2\alpha} \quad \text{— (I)}$$

Taking 2nd and last relation

$$-bp\,\mathrm{cosec}\,\varphi = (a^2-b^2)\sin\alpha$$

$$\Rightarrow \frac{-bp}{\sin\varphi} = (a^2-b^2)\sin\alpha$$

$$\Rightarrow \sin\varphi = \frac{-bp}{(a^2-b^2)\sin\alpha}$$

$$\Rightarrow \sin^2\varphi = \frac{b^2p^2}{(a^2-b^2)^2\sin^2\alpha} \quad \text{— (II)}$$

adding (I) and (II) we get

$$\cos^2\varphi + \sin^2\varphi = \frac{p^2}{(a^2-b^2)^2}\left[\frac{a^2}{\cos^2\alpha}+\frac{b^2}{\sin^2\alpha}\right]$$

$$\therefore \frac{a^2}{\cos^2\alpha}+\frac{b^2}{\sin^2\alpha} = \frac{(a^2-b^2)^2}{p^2}$$

is required condition

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