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PHYSICS (H)-PAPER I

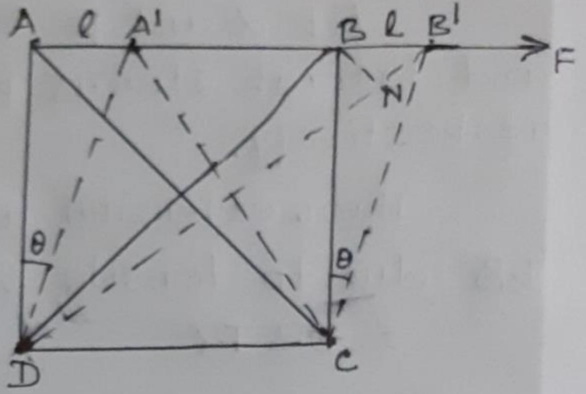
(ELASTICITY-II)

5

Elasticity — II

Elastic Constants

(ii) Relations between Y, η and σ :-



Let ABCD represent the front face of a cube of side L. A tangential force F is applied on its upper face AB and the bottom face DC is fixed. As a result of this force the cube is sheared to A'B'CD through an angle θ.

Then ~~shearing~~ shearing strain

$$\theta = \frac{AA'}{AD} = \frac{BB'}{BC} = \frac{l}{L}$$

Where the displacement $AA' = BB' = l$

Shearing stress

$$T = \frac{F}{\text{Area of the upper face of the cube}}$$

$$= \frac{F}{L^2}$$

$\therefore$ Coefficient of rigidity $\eta = \frac{T}{\theta}$

6

But a shearing stress along AB is equivalent to a tensile stress along DB and an equal compressive stress along AC at right angles to each other.

Let α and β be the longitudinal and lateral strains per unit stress respectively..

Then extension along diagonal DB due to tensile stress

$= DB\, T\alpha$

and extension along diagonal DB due to compression stress along AC

$= DB\, T\beta$

Total extension along DB

$= DB\, T(\alpha+\beta)$

$= L\sqrt{2}\, T(\alpha+\beta)$

$DB = \sqrt{L^2+L^2} = \sqrt{2L^2} = L\sqrt{2}$

Let ~~ab~~ us draw a perpendicular BN on DB'. Then increase in the length of diagonal DB is practically equal to NB'. As θ is very small therefore angle AB'C is nearly 90° and $\angle BB'N = \angle 45^0$

Thus $NB' = BB' \cos 45^0 = \frac{BB'}{\sqrt{2}} = \frac{l}{\sqrt{2}}$

7.

$\therefore L\sqrt{2}\, T(\alpha+\beta) = 4/\sqrt{2}$

or, $T\frac{L}{l} = \frac{1}{2(\alpha+\beta)}$

But $T\frac{L}{l} = \frac{T}{l/L} = \frac{T}{\theta} = \eta$

$\therefore \eta = \frac{1}{2(\alpha+\beta)} = \frac{1}{2\alpha(1+\beta/\alpha)}$

But $\beta/\alpha = \sigma$ and $Y = \frac{\text{Stress}}{\text{longitudinal strain}}$

$= \frac{1}{\alpha}$

Therefore $\boxed{\eta = \frac{Y}{2(1+\sigma)}}$ —— [B]

[III] <u>Relation between Y, K and η</u> :-

From ~~for~~ eqn [A]

$K = \frac{Y}{3(1-2\sigma)}$

or, $1-2\sigma = \frac{Y}{3K}$

FROM eqn [B]

$\eta = \frac{Y}{2(1+\sigma)}$

or, $2+2\sigma = \frac{Y}{\eta}$

8.

Adding the above two equations, We get

$$(1-2\sigma)+(2+2\sigma) = \frac{Y}{3K}+\frac{Y}{\eta}.$$

$$\text{or } 3 = \frac{Y}{3K}+\frac{Y}{\eta}.$$

$$\text{or } 3 = Y\left(\frac{1}{3K}+\frac{1}{\eta}\right)$$

$$\text{or } 3 = Y\left(\frac{\eta+3K}{3K\eta}\right)$$

Thus, $Y = \dfrac{9K\eta}{\eta+3K}$

This may be written as

$$\frac{9}{Y} = \frac{\eta+3K}{K\eta} = \frac{1}{K}+\frac{3}{\eta}$$

$$\text{or, } \boxed{\frac{9}{Y} = \frac{1}{K}+\frac{3}{\eta}} \quad \text{— [C]}$$

[IV] Relation between K, η and σ :-

From eqn. [A]

$$K = \frac{Y}{3(1-2\sigma)}$$

$$\text{or, } Y = 3K(1-2\sigma) \text{ — (a)}$$

From eqn [B]

$$\eta = \frac{Y}{2(1+\sigma)} \quad \text{or, } Y = 2\eta(1+\sigma) \text{ — (b)}$$

9

Equating the two values of Y (from eqn (a) and (b))

We have

$$3K(1-2\sigma) = 2\eta(1+\sigma)$$

or, $3K - 6K\sigma = 2\eta + 2\eta\sigma$

or, $3K - 2\eta = 2\eta\sigma + 6K\sigma$

or, $3K - 2\eta = \sigma(2\eta + 6K)$.

or, $$\boxed{\sigma = \frac{3K - 2\eta}{2\eta + 6K}} \quad \text{— [D]}$$