# VIBEKA NAND JHA

**ASSOCIATE PROFESSOR**

**DEPARTMENT OF PHYSICS**

**SNSRKS COLLEGE SAHARSA**


# PHYSICS (H)-PAPER I

## (ELASTICITY-IV)

4.

# Elasticity

## Expression for Bending Moment :-

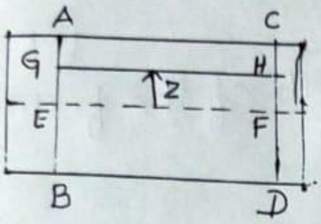


FIG - I

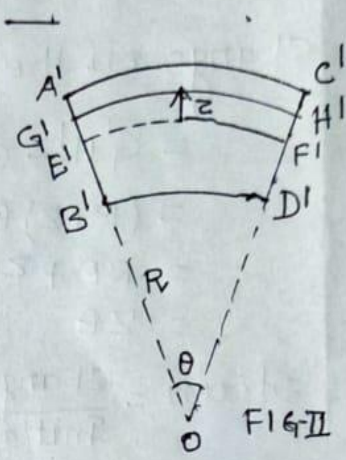


FIG-II

Let us consider a small portion of the beam bounded by two transverse sections AB and CD close to each other. (The weight of the beam has been neglected in comparision to the load W). After bending as shown in fig-II, AC is elongated to A'C' and BD is shortened to B'D'. Line EF represents the neutral surface, which is ~~neigther~~ neither stretched nor shortened.

Let the small portion, under condsideration be bent in the form of a circular arc, subtending and angle $\theta$ at the centre of curvature O. Let R be the radius of curvature of the neutral surface E'F'.

Consider a filament GH distant z from EF in the unbent position of the beam. After bending, its position is G'H' (fig-II)

Now, $G'H' = (R+z)\theta$

5.

Before bending $GH = EF = E'F'$

But $E'F' = R\theta$

$\therefore GH = R\theta$

[arc length $= \theta R$] Angle Radius

(If $\theta$ is in radians)

Change in the length of the filament

$$= G'H' - GH$$
$$= (R+Z)\theta - R\theta$$
$$= R\theta + Z\theta - R\theta$$
$$= Z\theta$$

$$\text{Strain} = \frac{\text{Change in length}}{\text{Initial length}}$$

$$= \frac{G'H' - GH}{GH} = \frac{Z\theta}{R\theta} = \frac{Z}{R} \quad \text{— [A]}$$

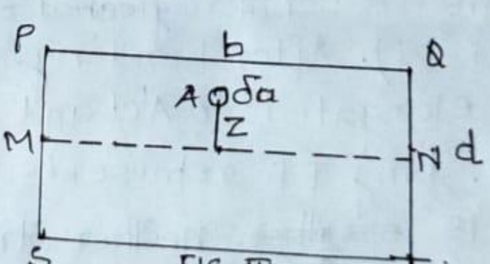


FIG - III

If PQRS represents a Section of the beam at right angles to its length and the plane of bending, then, clearly, the forces acting on the filaments are perpendicular to this Section. The line MN lies on the neutral Surface.

Let the breadth of the Section $PQ = b$ and its depth $QR = d$

The forces producing elongations and Contractions in filaments act perpendicularly to the upper and lower halves, 6

PQMN and MNRS repectively of the Section PQRS and their directions being opposite to each Other.

Now Consider a small area $\delta a$ of the section PQRS about a point A, distant Z from the neutral surface. The strain produced in a filament passing through this area will be $\frac{Z}{R}$ as ~~seen~~ shown above.

Now,
$$Y = \frac{\text{Stress}}{\text{Strain}}$$

or, Stress = Y x Strain

Hence ~~φ~~ Stress about the point $A = Y \times \frac{Z}{R}$

Force on the area $\delta a = Y \frac{Z}{R} \delta a$

Momentum of this force ~~about~~ about the line $MN = Y \cdot \frac{Z}{R} \delta a Z$

$$= \frac{YZ^2 \delta a}{R}$$

But the moments of the forces acting on both the upper and the lower halves of the Section are in the Same direction, therefore the total moment of the forces acting on the filaments in the section PQRS is

$$\Sigma \frac{Y \delta a z^2}{R} = \frac{Y}{R} \Sigma \delta a z^2 = \frac{YI}{R} \quad -[B]$$

Where $I = \Sigma \delta a z^2$ = Second moment of the Sectional area about MN and therefore equal to ~~φ~~ $ak^2$.

7

Where $a$ = Whole area of surface PQRS and $K$ = radius of gyration about MN.

The quantity $\frac{YI}{R}$ is the restoring couple or the bending moment of the beam.

Thus

$$\boxed{\text{Bending Moment} = \frac{YI}{R}}$$

The quantity $YI$ is known as flexural rigidity.