Probability.

If an event can happen in "a" ways and fail in "b" ways and all these ways are equally likely to occure, then the prob. of its happening is $\frac{a}{a+b}$ and that of its failing is $\frac{b}{a+b}$.

Thus if p denotes the prob of the happening of an event then $p = \frac{a}{a+b}$

and if q is the prob. of its not happening then,

$$q = \frac{b}{a+b}$$

Thus; $p+q = \frac{a}{a+b} + \frac{b}{a+b}$

$$= \frac{a+b}{a+b} = 1$$

Hence; $p = \frac{\text{favourable cases}}{\text{Total no. of cases}}$

Random variable :- By a random variable (r.v) we mean a real number X Connected with the outcome of a random experiment E. for example :- If E consists of two tosses of a coin we may consider the random variable which is the number of heads (0, 1 or 2)

outcome :- HH, HT, TH, TT
value of X :- 2, 1, 1, 0

Discrete random variable :- If a random variable takes at most a countable number of values, it is called a discrete random variable.

Continuous random variable :- A random variable X is said to be continuous if it can take all possible values between certain limits trials in which two events E_1 and E_2 are to happen.

Let P_1 is the prob. of event E_1; it will happen on P_1N occasions. Out of these P_1N occasions E_2 will happen on $P_2(P_1N) = P_1P_2N$ occasions.

Therefore the compound prob. that both of these happen $= \lim_{N\to\infty} \frac{P_1P_2N}{N}$

$= P_1 P_2$.

Generalisation:→ If there are n independent events whose respective prob. are $P_1, P_2 --- P_n$ then the compound prob. that all these happen $= P_1 P_2 --- P_n$.

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Multiplication theorem of expectation:- The mathematical expectation of the product of a number of independent random variables is equal to the product of their expectation.

Symbolically, if X, Y, Z ... T are n independent random variables then –

$$E(XYZ \cdots T) = E(X) \cdot E(Y) \cdots E(T).$$

Proof :- $E(XY) = \sum_{i=1}^{n} \sum_{j=1}^{m} x_i y_j P_{ij}$

$$= \sum_{i=1}^{n} \sum_{j=1}^{m} x_i y_j P_i P_j$$

$$= \sum_{i=1}^{n} x_i P_i \cdot \sum_{j=1}^{m} y_j \cdot P_j$$

$$= E(X) \cdot E(Y) \text{ Proved}$$

Symbolically; If X, Y, Z --- T are n random variables then

$$E(X+Y+Z+ \cdots +T) = E(X)+E(Y)+E(Z)+ \cdots +E(T)$$

Proof:→ Let the random variable X assumes the values x_1, x_2 --- x_n with prob. P_1, P_2 --- P_n, then by def^n;

$$E(X) = \sum_{i=1}^{n} P_i X_i$$

$$\text{Illy, } E(Y) = \sum_{j=1}^{m} P_j Y_j$$

$$\therefore E(X+Y) = \sum_i^n \sum_j^m P_{ij}(X_i + Y_j) = \sum P_i X_i + \sum P_j Y_j$$

$$= \sum_i^n \sum_j^m P_{ij} x_i + \sum_i^n \sum_j^m P_{ij} Y_j$$

$$= \sum_i^n \left(x_i \sum_j^m P_{ij}\right) + \sum_j^m \left(Y_j \sum_{i=1}^{n} P_{ij}\right)$$

$$= \sum_{i=1}^{n} x_i P_i + \sum_j^m Y_j P_j$$

$$= E(X) + E(Y) \quad \text{Proved}$$

Mathematical Expectation:→

If X is a discrete random variable which can assume any one of the values $x_1, x_2, x_3 \text{---} x_n$ with respective probabilities $p_1, p_2, p_3 \text{---} p_n$ then its mathematical expectation is defined as:—

$$E(X) = \sum_{i=1}^{n} p_i x_i \; ; \sum_{i=1}^{n} p_i = 1$$

On the other hand if X can take any one of the values $x_i; i = 1, 2 \text{---}$ with respective prob. p_i then —

$$E(X) = \sum_{i=1}^{\infty} p_i x_i \; ; \sum_{i=1}^{\infty} p_i = 1$$

Addition theorem of expectation

The mathematical expectation of the sum of random variables is equal to the sum of their expectation.

Mutually exclusive events :- Events are called mutually exclusive if one of them happens other can not happen.
example :→. In tossing a coin if a head occure, tail can not occure. Thus these are mutually exclusive events. etc.
Independent Events →. If two or more events occure without affecting one another they are said to be independent events. Example :→. In tossing a coin the event of getting a head in the first toss is independent of getting a head in second, third and subsequent throws. etc
Compound probability :→ (Multiplication theory of prob.) :→.
Let N be the number of

Sample space :- The totality of all possible outcomes of a random experiment constitutes the Sample space.

The experiment is known as a trial and the outcomes are known as events or Cases.

Example :- Tossing of a coin is a trial and getting head or tail is an event. Throwing of a die is a trial and getting 1 (or 2, 3 -- 6) is an event. etc.

Equally likely Event :→

Two events are considered equally likely if one of them cannot be expected in preference to the other.

example :- In tossing of a coin head or tail are equally likely vents etc.