$\therefore \mu_1' = np$ (mean)

Variance :→

$\because \mu_2' = E(x^2)$

$= \sum_{x=0}^{n} x^2 \cdot \binom{n}{x} \cdot p^x \cdot q^{n-x}$

$= \sum_{x=0}^{n} \{x(x-1)+x\} \cdot \frac{n(n-1)}{x(x-1)} p^x q^{n-x}$

$= n(n-1)p^2 \cdot \left[\sum_{x=2}^{n} \binom{n-2}{x-2} p^{x-2} q^{n-x}\right] + np$

$\mu_2' = n(n-1)p^2 + np$

$\therefore \mu_2 = \mu_2' - \mu_1'^2$

$= n(n-1)p^2 + np - (np)^2$

$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$

$= np(1-p)$

$= npq$ variance

gf is applicable to find the prob. generating f^n; moment generating

$= \lambda^2 + \lambda.$

$\therefore \mu_2 = \mu_2' - \mu_1'^2$

$= \lambda^2 + \lambda - \lambda^2 = \lambda.$

i.e. Mean & variance are equal for poisson dist.

Property: → Sum of independent poisson variates is also a poisson variate. But difference of two independent poisson variates is not a poisson variate.

Poisson dist. may be applicable to the following cases: →

(i) Number of deaths from a disease such as heart attack, or cancer or due to ~~Sna~~ snake bite.

(ii) Number of suicide reported in a particular city.

(iii) Number of defective material in a packing manufactured by a good concern. etc.

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function, Characterstic fun, Cumulants and so on. It is also useful to fit any distribution.

Poisson Dist:→ Poisson dist. was discovered by the French mathematician and physicist Simen Denis Poisson (1781 – 1840) who published it in 1837. Poisson dist. is a limiting case of binomial dist. under the following Condition →

(i) when $n \to \infty$; (ii) $p \to 0$ and (iii) $np = \lambda$

Defn:→ A random variable x is said to follow a poisson dist. if it assumes only non-negative values and its prob. mass fn is given by: —

$$P(x,\lambda) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!} \; ; \; x = 0, 1, 2 \ldots$$

Here λ is known as the parameter of the dist. and $\lambda > 0$.

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Normal Distribution: → The normal dist. was first discovered in 1733 by English mathematician De-Moivre who obtained this continuous dist. as a limiting case of the binomial dist. The normal model has the most important probability model in statistical analysis.

Defn: → A random variable x is said to have a normal dist. with parameters μ (called mean) and σ^2 (called variance) if its p density fn is given by the prob. law: →

$$f(x:\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$-\infty < x < \infty.$$

$$-\infty < \mu < \infty, \ \sigma > 0.$$

It has the following properties:−

(i) The curve is bell shaped and symmetrical about the line $x = \mu$.

$x = \mu$

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(ii) Mean, Median & Mode of the dist. coincide.

(iii) Linear Combination of independent normal variates is also a normal variate.

(iv) X-axis is an asymptote to the curve.

from normal integral table we conclude the area property:

$P(\mu - \sigma < X < \mu + \sigma) = 0.6826$.

$P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.9544$.

$P(\mu - 3\sigma < X < \mu + 3\sigma) = 0.9973$.

Importance of the normal dist: →

Normal dist. plays a very important role in statistical theory because of the following reason: →

(i) Most of the dist. e.g. Binomial, Poisson, Hypergeometric etc. can be approximated by normal dist.

Many of the sampling dist. tend to normality for large samples.

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(ii) The proofs of all the tests of significance in sampling are based upon the fundamental assumption that the population from which the samples have been drawn is normal.

(iii) Normal dist. finds large application in statistical quality control.

Proof of area property :-

$$P(-1<z<1) = \int_{-1}^{1} \phi(z)\,dz = 2\int_{0}^{1} \phi(z)\,dz$$

$$= 2 \times .3413 = .6826$$

$$P(-2<z<2) = \int_{-2}^{2} \phi(z)\,dz = 2\int_{0}^{2} \phi(z)\,dz$$

$$= 2 \times .4772 = .9544$$

$$P(-3<z<3) = \int_{-3}^{3} \phi(z)\,dz$$

$$= 2\int_{0}^{3} \phi(z)\,dz$$

$$= 2 \times .49865 = .9973.$$

Mean:→ ∵ $\mu_1' = E(X) = \sum_{x=0}^{\infty} x \cdot P(x,\lambda)$

$= \sum_{x=0}^{\infty} x \cdot \frac{e^{-\lambda}\lambda^x}{x!}$

$= \lambda \cdot e^{-\lambda} \sum_{x=1}^{\infty} \left(\frac{\lambda^{x-1}}{(x-1)!}\right)$

$= \lambda \cdot e^{-\lambda} \left\{1 + \lambda + \frac{\lambda^2}{\lfloor 2} + \frac{\lambda^3}{\lfloor 3} + \cdots\right\}$

$= \lambda \cdot e^{-\lambda} e^{\lambda} = \lambda$ (Mean)

Variance:→ $\mu_2' = E(X^2)$

$= \sum_{x=0}^{\infty} x^2 \cdot P(x,\lambda)$

$= \sum_{x=0}^{\infty} \{x(x-1) + x\} \cdot \frac{e^{-\lambda}\lambda^x}{x!}$

$= e^{-\lambda} \sum_{x=0}^{\infty} x(x-1) \frac{\lambda^x}{x!} +$

$\sum_{x=0}^{\infty} x \cdot \frac{e^{-\lambda}\lambda^x}{x!}$

$= \lambda^2 \cdot e^{-\lambda} \sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!} + \lambda$

$= \lambda^2 \cdot e^{-\lambda} \cdot e^{\lambda} + \lambda.$