$$\therefore \frac{\Sigma d_i^2}{n} = 2\sigma_x^2 - 2r\sigma_x^2 = 2\sigma_x^2(1-r)$$

$$\text{or, } 1-r = \frac{\Sigma d_i^2}{n \cdot 2\sigma_x^2} = \frac{\Sigma d_i^2}{n \cdot 2 \cdot \frac{(n^2-1)}{12}}$$

$$\text{or, } 1-r = \frac{6\Sigma d_i^2}{n(n^2-1)}$$

$$\text{or, } r = 1 - \frac{6\Sigma d_i^2}{n(n^2-1)} \quad \text{Ans}$$

Problem :- find rank corr. of

1) X :- 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14,
Y :- 1, 10, 3, 4, 5, 7, 2, 6, 8, 11, 15, 9, 14, 12
X :- 15, 16.
Y :- 16, 13. {Ans 1 - r = 0.8.}

2) X :- 1, 6, 5, 10, 3, 2, 4, 9, 7, 8.
Y :- 3, 5, 8, 4, 7, 10, 2, 1, 6, 9. $\left\{ -\frac{7}{33} \right.$

Proof: $\because \bar{x} = \bar{y} = \frac{n+1}{2}$

$$\sigma_x^2 = \sigma_y^2 = \frac{1}{n}\sum x_i^2 - \bar{x}^2$$

$$= \frac{1}{n}(1^2+2^2+\dots+n^2) - \left(\frac{n+1}{2}\right)^2$$

$$= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n+1}{2}\right)^2$$

$$= \frac{n^2-1}{12}.$$

$\because d_i = x_i - y_i$

$$= (x_i - \bar{x}) - (y_i - \bar{y}).$$

Squaring and summing over i from 1 to n;

$$\frac{\sum d_i^2}{n} = \frac{\sum\{(x_i-\bar{x})-(y_i-\bar{y})\}^2}{n}$$

$$= \frac{\sum(x_i-\bar{x})^2}{n} + \frac{\sum(y_i-\bar{y})^2}{n} - \frac{2\sum(x_i-\bar{x})(y_i-\bar{y})}{n}$$

$$= \sigma_x^2 + \sigma_y^2 - 2\,\mathrm{cov}(X,Y)$$

$$= \sigma_x^2 + \sigma_y^2 - 2\cdot r\cdot\sigma_x\cdot\sigma_y.$$

Rank-corr. :→ Let us suppose that a group of n individuals is arranged in order of merit or proficiency. These ranks in the two characteristics will be different. e.g. if we consider the relation between intelligence and beauty, it is not necessary that a beautiful candidate is intelligent also.

Let (x_i, y_i) $i = 1, 2 \ldots n$ be the ranks of the ith individual in two characteristics. Pearsonian coeft. of correlation between the ranks x_i and y_i is called the rank corr. coeft. and is given by the formulas

$$r = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)} \quad \left\{ \begin{array}{l} \text{Spearman's} \\ \text{formula.} \end{array} \right.$$

29 finite difference

The value of the independent variable is called the argument and value of dependent vato variable corresponding to that argument is called the entry.

The difference between the consecutive value of the variable is called the interval of differencing.

Let the values of x are $f(a), f(a+h), f(a+2h), f(a+3h)$ ----

Then →

$$\Delta f(a) = f(a+h) - f(a)$$
$$\Delta f(a+h) = f(a+2h) - f(a+h)$$
$$\Delta f(a+2h) = f(a+3h) - f(a+2h)$$

etc.

These are called first difference.

Illy; second differences are

$$\Delta^2 f(a) = \Delta f(a+h) - \Delta f(a)$$
$$= f(a+2h) - f(a+h) - f(a+h) + f(a)$$
$$= f(a+2h) - 2f(a+h) + f(a)$$
etc.

~~Sil~~ Illy; we can obtained third differences, 4th differences and so on.

All these differences are summerise in a table called difference table.

x Argument	Y entry	1st diff. $\Delta f(x)$	2nd diff $\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
3·0	·33333				
		−·01075			
3·1	·32258		·00067		
		−·01008		−·00006	
3·2	·31250		·00061		[illegible]
		−·00947		−·00005	
3·3	·30303		·00056		
		−·00891			
3·4	·29412				

The first differences are obtained by subtracting each entry from the next entry. The second differences are obtained by carrying out a similar set of subtractions on the first differences and so on.

Task:- Construct the difference table:-

Argument:-	3.60,	3.61,	3.62,	3.63,	3.64,	3.65,	3.66,	3.67,	3.68.
Entry:-	.112046,	.120204,	.128350,	.136462,	.144600,	.152702,	.160788,	.168857,	.176908.

Given $u_0 = 3$, $u_1 = 12$, $u_2 = 81$, $u_3 = 200$, $u_4 = 100$ and $u_5 = 8$. find $\Delta^5 u_0$

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Newton Gregory forward formula :-

$$P_n(x) = f(a) + u\Delta f(a) + \frac{u(u-1)}{2!}\Delta^2 f(a) + \frac{u(u-1)(u-2)}{\lfloor 3}\Delta^3 f(a) + \cdots + \frac{u(u-1)(u-2)\cdots(u-n+1)}{\lfloor n}\Delta^n f(a)$$

where $u = \frac{x-a}{h}$

This formula is used mainly for interpolating the values of $f(x)$ near the begining of a set of tabulated values.

Interpolation :→ Interpolation means insertion or filling up intermediate terms of a series. It is the technique ~~of~~ of estimating the value of a function for any intermediate value of the independent variable when the values ~~to~~ of the fn corresponding to a number of the values of the variable are given.

Extrapolation :→ The process of computing the value of a fn outside the range of given values of the variable is called ~~exp~~ extrapolation

∇ :- The backward difference usually denoted by ∇ are defined as:-
$\nabla f(a+h) = f(a+h) - f(a)$
$\nabla^2 f(a+2h) = \nabla f(a+2h) - \nabla f(a+h)$

Newton Gregory backward formula :-

$$P_n(x) = f(a) + u\nabla f(a) + \frac{u(u+1)}{2!}\nabla^2 f(a) + \frac{u(u+1)(u+2)}{3!}\nabla^3 f(a) + \cdots + \frac{u(u+1)(u+2)+\cdots(u+n-1)}{n!}\Delta^n f(a).$$

This formula is used for interpolating the values of $f(x)$ near the end of the tabular values.

Lagranges formula →

$$P_n(x) = \frac{(x-x_1)(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\cdots(x_0-x_n)} f(x_0)$$

$$+ \frac{(x-x_0)(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\cdots(x_1-x_n)} f(x_1)$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)\cdots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\cdots(x_2-x_n)} f(x_2)$$

$$+ \cdots$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\cdots(x_n-x_{n-1})} f(x_n)$$

(1) find value of Y when
x = 8

x :-	5	9	11	12	65
Y :	121	73	25	16	

(2) find x = 27

x: 14, 17, 31, 35
Y: 68.7 64.0, 44.0, 39.1 | 50.04

(3) find the missing ter

x: 1, 2, 3, 4, 5, 6, 7
Y: 2, 4, 8, ?, 32, 64, 128 | 16.1

(4) X: 1 2 3 4 5
Y: 7 ? 13 21 37 [9.5]

(5) X:- 1 2 3 4 5
Y:- 2 5 7 ? 32

* The operator E is defined as increasing the argument by the interval of differencing.

Operator Δ and E :→

Let the fn of x and let $a, a+h, a+2h, ---$ be the consecutive values of x.

Then; $\Delta f(a) = f(a+h) - f(a)$

* we define an operator E called the displacement operator by the equn :-

$$Ef(a) = f(a+h)$$

Relation :- $\Delta f(a) = f(a+h) - f(a)$

So, $\Delta f(a) = Ef(a) - f(a)$

So, $Ef(a) = f(a) + \Delta f(a)$

$$Ef(a) = (1+\Delta) f(a)$$

$$\left.\begin{aligned} \text{So, } E &= 1+\Delta \\ \text{So, } \Delta &= E-1. \end{aligned}\right\}$$