Ex $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ find $A^2 - 4A + 3I$ 01

Now $A^2 = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4+1 & -2-2 \\ -2-2 & 1+4 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$

$\therefore A^2 - 4A + 3I = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} - 4\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} - \begin{bmatrix} 8 & -4 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} 8 & -4 \\ -4 & 8 \end{bmatrix} - \begin{bmatrix} 8 & -4 \\ -4 & 8 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ Ans

$= 0$ Ans

Ex $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 2 & 4 \end{bmatrix}$

then prove that $(AB)' = B'A'$

Now $AB = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 2 & 4 \end{bmatrix}$

$= \begin{bmatrix} 1+2+6 & 3-0+12 \\ -4-2+10 & -12+0+20 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 4 & 8 \end{bmatrix}$

$\therefore (AB)' = \begin{bmatrix} 9 & 4 \\ 15 & 8 \end{bmatrix}$ —— (1)

Again $B' = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 4 \end{bmatrix}$, $A' = \begin{bmatrix} 1 & -4 \\ -2 & 2 \\ 3 & 5 \end{bmatrix}$

$B'A' = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 4 \end{bmatrix}\begin{bmatrix} 1 & -4 \\ -2 & 2 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 1+2+6 & -4-2+10 \\ 3+0+12 & -12+0+20 \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ 15 & 8 \end{bmatrix}$ —(II)

from (I) & (II) we get

$(AB)' = B'A'$ proved

(Ex) Express $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix}$ as the sum of symmetric & sk

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix}$ $\therefore A' = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}$

$\therefore A + A' = \begin{bmatrix} 2 & 5 & 8 \\ 5 & 8 & 11 \\ 8 & 11 & 14 \end{bmatrix}$

& $A - A' = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

$\therefore$ Symmetric matrix X (let) $= \dfrac{A+A'}{2}$

$= \dfrac{1}{2}\begin{bmatrix} 2 & 5 & 8 \\ 5 & 8 & 11 \\ 8 & 11 & 14 \end{bmatrix}$

& skew sym. matrix Y (say) $= \dfrac{1}{2}(A - A')$

$= \dfrac{1}{2}\begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

(B) Obtain the reciprocal matrix of the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$

$\therefore$ Reciprocal of $A = \frac{1}{A} = A^{-1} = \frac{\text{Adj } A}{|A|}$ (Formula)

Now $|A| = \begin{vmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{vmatrix}$

$= 1(-28+30) - 3(0-6) + 0$

$= 2 + 18 = 20 \neq 0$

Now Co-factor of a_{11} i.e. $1 = (-28+30) = 2$

______ a_{12} i.e. $0 = -(-21-0) = 21$

______ a_{13} i.e. $-1 = (-18-0) = -18$

______ a_{21} i.e. $3 = -(0-6) = 6$

______ a_{22} i.e. $4 = (-7+0) = -7$

______ a_{23} i.e. $5 = -(-6-0) = 6$

______ a_{31} i.e. $0 = (0+4) = 4$

______ a_{32} i.e. $-6 = -(5+3) = -8$

______ a_{33} i.e. $-7 = (4-0) = 4$

$\therefore$ Co-factors of $A = B$ (say) $= \begin{bmatrix} 2 & 21 & -18 \\ 6 & -7 & 6 \\ 4 & -8 & 4 \end{bmatrix}$

$\therefore$ Adj $A = B^{T} = \begin{bmatrix} 2 & 6 & 4 \\ 21 & -7 & -8 \\ -18 & 6 & 4 \end{bmatrix}$

$\therefore$ Reciprocal of $A = \frac{\text{adj } A}{|A|} = \frac{1}{20} \begin{bmatrix} 2 & 6 & 4 \\ 21 & -7 & -8 \\ -18 & 6 & 4 \end{bmatrix}$

$= \begin{bmatrix} \frac{2}{20} & \frac{6}{20} & \frac{4}{20} \\ \frac{21}{20} & \frac{-7}{20} & \frac{-8}{20} \\ \frac{-18}{20} & \frac{6}{20} & 4/20 \end{bmatrix}$ Ans

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