$$\Rightarrow e^2 + 2e\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} + 2\cos^2\frac{\alpha-\beta}{2} - 1 = 0 \quad \text{--- (viii)}$$

To get the equation of the director circle, we have to eliminate α and β from (iii), (iv), (v) and (viii).

Thus from (iii) and (v) we have

$$\frac{l}{r_1} = e\cos\theta_1 + \cos\frac{\alpha-\beta}{2}$$

$$\Rightarrow \cos\frac{\alpha-\beta}{2} = \frac{l}{r_1} - e\cos\theta_1$$

$\therefore$ from (viii) we have

$$e^2 + 2e\cos\theta_1\left(\frac{l}{r_1} - e\cos\theta_1\right) + 2\left(\frac{l}{r_1} - e\cos\theta_1\right)^2 - 1 = 0$$

$$\Rightarrow e^2 + 2\frac{l}{r_1}e\cos\theta_1 - 2e^2\cos^2\theta_1 + 2\left(\frac{l^2}{r_1^2} - 2\frac{l}{r_1}e\cos\theta_1 + e^2\cos^2\theta_1\right) - 1 = 0$$

$$\Rightarrow e^2 + \frac{2l}{r_1}e\cos\theta_1 - \cancel{2e^2\cos^2\theta_1} + \frac{2l^2}{r_1^2} - \frac{4l}{r_1}e\cos\theta_1 + \cancel{2e^2\cos^2\theta_1} - 1 = 0$$

$$\Rightarrow \frac{2l^2}{r_1^2} - \frac{2l\,e\cos\theta_1}{r_1} + e^2 - 1 = 0$$

$$\Rightarrow r_1^2(1 - e^2) + 2l e r_1\cos\theta_1 - 2l^2 = 0$$

Hence the locus of (r_1, θ_1) is

$$r^2(1-e^2) + 2ler\cos\theta - 2l^2 = 0$$

which is the required equation of director circle.

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11. Find the equation of Director circle of the conic $\frac{l}{r} = 1 + e\cos\theta$

Soln:- The equations of the tangents at the pts. on the conic whose vectorial angles are α and β are

$\frac{l}{r} = e\cos\theta + \cos(\theta - \alpha)$ ——— (I)

and $\frac{l}{r} = e\cos\theta + \cos(\theta - \beta)$ ——— (II)

if these tangents interdict at the pt. (r_1, θ_1)

we have $\frac{l}{r_1} = e\cos\theta_1 + \cos(\theta_1 - \alpha)$ ——— (III)

and $\frac{l}{r_1} = e\cos\theta_1 + \cos(\theta_1 - \beta)$ ——— (IV)

Solving (III) and (IV) we get

$\cancel{e\cos\theta_1} + \cos(\theta_1 - \alpha) = \cancel{e\cos\theta_1} + \cos(\theta_1 - \beta)$

$\Rightarrow \cos(\theta_1 - \alpha) = \cos(\theta_1 - \beta)$

$\Rightarrow \theta_1 - \alpha = -(\theta_1 - \beta) \quad (\because \alpha \neq \beta)$

$\Rightarrow 2\theta_1 = \alpha + \beta$

$\therefore \theta_1 = \frac{\alpha + \beta}{2}$ ——— (V)

Transforming the equation of the tangent (I) to Cartesian Co-ordinates, we get

$l = er\cos\theta + r(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$

$= (e + \cos\alpha)r\cos\theta + \sin\alpha \cdot r\sin\theta$

$= (e + \cos\alpha)x + y\sin\alpha$ ——— (VI)

similarly (II) is

$l = (e + \cos\beta)x + y\sin\beta$ ——— (VII)

$\because$ tangents are at right angles, we have from (VI) and (VII)

$(e + \cos\alpha)(e + \cos\beta) + \sin\alpha \cdot \sin\beta = 0$

$\Rightarrow e^2 + e(\cos\alpha + \cos\beta) + \cos\alpha\cos\beta + \sin\alpha\sin\beta = 0$

$\Rightarrow e^2 + e \cdot 2\cos\frac{\alpha+\beta}{2} \cdot \cos\frac{\alpha-\beta}{2} + \cos(\alpha - \beta) = 0$