

Th-2. Find the condition that the line $(x+my)+n=0$ may be a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Proof:- If $(x+my)+n=0$ be normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ then it must be identical with the equation

$$\frac{ax}{\cos\phi} - \frac{by}{\sin\phi} = a^2 - b^2$$

the condition for this is

$$\frac{x}{a} = \frac{m}{-\frac{b}{\sin\phi}} = \frac{n}{-(a^2 - b^2)}$$

$$\Rightarrow \frac{\cos\phi}{a} = -\frac{m \sin\phi}{b} = \frac{n}{-(a^2 - b^2)}$$

$$\therefore \cos\phi = -\frac{an}{b(a^2 - b^2)}$$

$$\text{and } \sin\phi = \frac{bn}{m(a^2 - b^2)}$$

Now, squaring and adding, we get

$$\cos^2\phi + \sin^2\phi = \frac{a^2 n^2}{b^2 (a^2 - b^2)^2} + \frac{b^2 m^2}{m^2 (a^2 - b^2)^2}$$

$$\Rightarrow 1 = \frac{a^2 n^2}{b^2 (a^2 - b^2)^2} + \frac{b^2 m^2}{m^2 (a^2 - b^2)^2}$$

$$\Rightarrow \frac{a^2}{b^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)^2}{b^2}$$

which is required

Condition

Dr. G. S. Raut
Head, Dept. of Maths
J. J. Somaiya Institute of
Technical Education

Proof: Let the eqn of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
 We know from Differential Calculus that the Equation of tangent at the point (x₁, y₁) is given by the formula

$$x - y_1 = \left(\frac{dy}{dx} \right)_{(x_1, y_1)} (x - x_1) \quad \text{--- (1)}$$

∴ eqn of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
 D. w. r. t. x we get

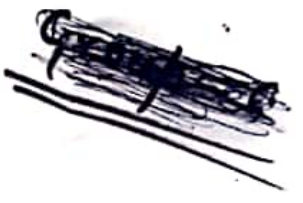
$$\frac{2x}{a^2} + \frac{2y}{b^2} \frac{dy}{dx} = 0$$

$$\frac{2}{a^2} + \frac{y}{b^2} \frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = - \frac{b^2 x}{a^2 y}$$

at point (x₁, y₁)

$$\frac{dy}{dx} = - \frac{b^2 x_1}{a^2 y_1}$$



Hence from (1) we have

$$x - y_1 = - \frac{b^2 x_1}{a^2 y_1} (x - x_1)$$

$$\Rightarrow a^2 y_1 - b^2 x_1 = - \frac{b^2 x_1}{a^2 y_1} (a^2 x - a^2 x_1)$$

$$\Rightarrow b^2 a x_1 + a^2 y_1 = b^2 x_1 + a^2 y_1$$

Dividing both sides by $a^2 b^2$ we get

$$\frac{a x_1}{a^2} + \frac{y_1}{b^2} = \frac{x_1}{a^2} + \frac{y_1}{b^2} \quad \text{--- (ii)}$$

(∵ Point (x₁, y₁) lies on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
 ∴ the pt. satisfies the eqn of ellipse

$$\text{Hence } \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} = 1$$

Thus equation (ii) becomes tangent at
 $\frac{a x_1}{a^2} + \frac{y_1}{b^2} = 1$ is required eqn of tangent at
 pt (x₁, y₁) to the ellipse