Th:- 2. Find the Condition that the line $lx + my + n = 0$ may be a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Proof :- If $lx + my + n = 0$ be normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ then it must be identical with the equation

$$\frac{ax}{\cos\varphi} - \frac{by}{\sin\varphi} = a^2 - b^2$$

the condition for this is

$$\frac{l}{\frac{a}{\cos\varphi}} = \frac{m}{-\frac{b}{\sin\varphi}} = \frac{n}{-(a^2-b^2)}$$

$$\Rightarrow \frac{l\cos\varphi}{a} = -\frac{m\sin\varphi}{b} = \frac{n}{-(a^2-b^2)}$$

$$\therefore \cos\varphi = -\frac{an}{l(a^2-b^2)}$$

and $\sin\varphi = \frac{bn}{m(a^2-b^2)}$

Now, squaring and adding, we get

$$\cos^2\varphi + \sin^2\varphi = \frac{a^2n^2}{l^2(a^2-b^2)^2} + \frac{b^2n^2}{m^2(a^2-b^2)^2}$$

$$\Rightarrow 1 = \frac{n^2}{(a^2-b^2)^2}\left[\frac{a^2}{l^2} + \frac{b^2}{m^2}\right]$$

$$\Rightarrow \frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2-b^2)^2}{n^2}$$

which is required Condition.

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2 Dim^n i.e. Co-ordinate Geom.

Th:- 1 Find the equation of tangent at the point (x_1, y_1) to the ellipse

Proof:- Let the eq^n of ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
We know from Differential Calculus that the Equation of tangent at the point (x_1, y_1) is given by the formula

$$y-y_1=\left(\frac{dy}{dx}\right)_{(x_1,y_1)}(x-x_1) \quad \text{—(I)}$$

$\because$ Eq^n of ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
D. w. r. t. x we get

$$\frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0$$

$$\frac{x}{a^2}+\frac{y}{b^2}\frac{dy}{dx}=0$$

$$\therefore \frac{dy}{dx}=-\frac{b^2x}{a^2y}$$

at point (x_1, y_1)

$$\frac{dy}{dx}=-\frac{b^2x_1}{a^2y_1}$$

Hence from (I) we have

$$y-y_1=-\frac{b^2x_1}{a^2y_1}(x-x_1)$$

$$\Rightarrow a^2yy_1-a^2y_1^2=-b^2xx_1+b^2x_1^2$$

$$\Rightarrow b^2xx_1+a^2yy_1=b^2x_1^2+a^2y_1^2$$

Dividing both sides by a^2b^2, we get

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2} \quad \text{—(II)}$$

($\because$ point (x_1, y_1) lies on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
so the pt. satisfies the eq^n of ellipse
Hence $\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}=1$

Thus equation (II) becomes

$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$ is required eq^n of tangent at pt (x_1, y_1) to the ellipse