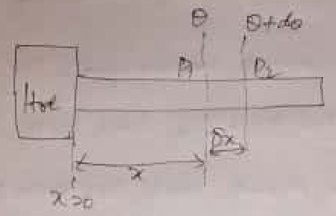
(5)

Rectilinear flow of heat along a Bar:-

(Fourier Equation of Heat flow) - Let us consider a bar of uniform area of cross-section A, which is heated at one end




Heat is flowing along the length of the bar. Let us consider two planes $P_1$ and $P_2$ perpendicular to the length of the bar at a distance $x$ and $x+\delta x$ from the hot end.

The temp. gradient at the plane $P_1 = \frac{d\theta}{dx}$

where $\theta$ is the excess of temp. (above the surroundings) of the bar at $P_1$. The excess of temp. at

$$P_2 = \theta + \frac{d\theta}{dx}\delta x$$

The temp. gradient at $P_2 = \frac{d}{dx}\left\{\theta + \frac{d\theta}{dx}\delta x\right\}$.

Heat flowing through $P_1$ in one second

$$Q_1 = -KA\frac{d\theta}{dx}.$$

Heat flowing through $P_2$ in one second

$$Q_2 = -KA\frac{d}{dx}\left\{\theta + \frac{d\theta}{dx}\delta x\right\}.$$

Heat gained per second by the rod between the planes $P_1$ and $P_2$

$$Q = Q_1 - Q_2$$

$$= -KA\frac{d\theta}{dx} + AK\frac{d}{dx}\left(\theta + \frac{d\theta}{dx}\delta x\right).$$

Hari Darshan Singh
Asso. Prof
Deptt. of physics
S N S R K S College Saharsa

D.P.I. Lecture No (3)

Transmission of Heat:-

Heat can be transferred from one place to the other by three different ways, i.e. Conduction, Convection and Radiation.

Conduction - In this process heat is transmitted from one point to another through the substance without the actual motion of the particles. In this case heat travels through the body from molecule to molecule from its hotter to its colder end, since molecules in a solid remain more or less fixed in their mean positions about which they can simply vibrate. In this process of heat transfer the molecules at the hotter end start vibrating vigorously with a large amplitude, and collide against those next to them and set them into similar vibrations. Each molecule thus transmits some of the heat (thermal energy) it received from its predecessor to its successor. This is the reason, why, if we place one end of the metal rod in the fire, the other end soon becomes hot.

and is known as Fourier's differential equation.

Special Cases -

(1) when heat lost by radiation is negligible -

This situation comes when the rod is completely covered by insulating materials,

It means, $C_p \cdot \delta x \cdot \theta = 0$.

$$a, \quad \frac{\partial^2 \theta}{\partial x^2} = \frac{\rho s}{k} \cdot \frac{d\theta}{dt}$$

$$= \frac{1}{h} \cdot \frac{d\theta}{dt}.$$

Here, $h = \frac{k}{\rho s}$, the thermal diffusivity of the rod.

Case (2) - After the steady state is reached:-

In this case, $\frac{d\theta}{dt} = 0$.

$\Rightarrow$ the rate of rise the temp. of the bar be zero.

Hence, $$\frac{d^2\theta}{dx^2} = \frac{C_p}{kA} \theta.$$

(3)

Although conduction is possible in the case of solid, liquid as well as in gases, yet it is usually the characteristic of the solids. There is very little conduction in liquids (except in mercury, Helium(II)) and gases.

Convection - It is the process in which heat is transmitted from one place to the other by the actual movement of the heated particles. It is prominent in the case of liquids and gases. Land and sea breezes and trade winds are formed due to convection.

There are two distinct types of convection (i) free or natural convection and (ii) forced convection.

Radiation - It is the process in which heat is transmitted from one place to the other directly without the ~~nece~~ necessity of the intervening medium. We get heat radiations directly from the Sun without affecting the intervening medium. Heat radiation can pass through vacuum. Their properties are similar to ~~to~~ light radiations.

Coefficient of thermal conductivity:

Let us consider a cube of side $x$ and area of each face $A$ sq.cm. The opposite faces of the cube are maintained at temp. $\theta_1$ and $\theta_2$, or, $\frac{d^2\theta}{dx^2} = \mu^2\theta$, where

we take here, $\frac{hp}{KA} = \mu^2$

The general solution of this equation is,

$$\theta = Ae^{+\mu x} + Be^{-\mu x}.$$

Where A and B are two unknown constants to be determined from the boundary conditions of the problem.

or, $Q = KA \frac{d^2\theta}{dx^2}\delta x$

Before the steady state is reached –

Before steady state, the quantity of heat Q is used in two ways: partly the heat is used to raise the temp. of the rod and the rest is lost due to radiation. The heat used per second to raise the temp of the rod

$$= \text{mass} \times \text{specific heat} \times \frac{d\theta}{dt}$$

$$= (A \cdot \delta x)\rho \cdot s \cdot \frac{d\theta}{dt}$$

where $\rho$ is the density of the material, $s$ is the specific heat, and $\frac{d\theta}{dt}$ is the rate of rise of the temp of the bar.

The heat lost per second due to radiation

$$= Ep\,\delta x \cdot \theta$$

where, $E$ is the emissive power of the surface and $p$ is the perimeter.

Now $Q = A\delta x\rho \cdot s \frac{d\theta}{dt} + Ep\,\delta x\,\theta$

or, $KA\frac{d^2\theta}{dx^2}\delta x = A\delta x\rho\, s\frac{d\theta}{dt} + Ep\,\delta x\,\theta$

or, $\frac{d^2\theta}{dx^2} = \frac{\rho s}{K}\frac{d\theta}{dt} + \frac{Ep}{KA}\theta$.

This is the general equation of the rectilinear flow of heat along a bar of uniform area of cross section.

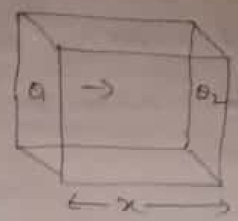
vity is defined as the amount of heat flowing in one second across the opposite faces of the cube of side one cm maintained at a difference of temperature of 1°C.

**Temperature gradient**:- The quantity $\frac{\theta_1 - \theta_2}{x}$ represents the rate of fall of temp. with respect to distance. The quantity $\frac{d\theta}{dx}$ represents the rate of change of temperature with respect to the distance. As temp. decreases with increase in distance from the hot end, the quantity $\frac{d\theta}{dx}$ is negative and is called the temp. gradient.

$$\therefore Q = -KA\left(\frac{d\theta}{dx}\right)t$$

It may be noted here that this relation holds ~~goods~~ good only when the steady state has ~~been~~ been reached. Steady or stationary state is that when no change of temperature along the rod.

where $\theta_1 > \theta_2$. heat is conducted in the direction of the fall of temp.




During the steady state, the quantity of heat Q flowing across the two opposite faces of an element is directly proportional to (i) the difference of temperature $\theta_1 - \theta_2$

(ii) the area of cross-section A

(iii) the time t for which heat is allowed to flow through it

& (iv) inversely proportional to the thickness (side).

i.e. $Q \propto \frac{A(\theta_1-\theta_2)}{x}$ i.e, $Q \propto \frac{A(\theta_1-\theta_2)}{x} t$

or, $Q = K \cdot \frac{A(\theta_1-\theta_2)t}{x}$

Here, K is the constant called the co-efficient of thermal conductivity of the material of the cube. If

$A = 1$ sq.cm.

$\theta_1 - \theta_2 = 1°C$

$t = 1s$

$x = 1$ cm

then $\boxed{Q = K}$

Therefore, the co-efficient of thermal conducti-